

# Infrared stability of quarkyonic matter with the $1/p^4$ confinement

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We demonstrate an exact cancellation of the infrared divergences in the color-singlet quarkyonic matter with the  $1/\bar{p}^4$  Coulomb-like confining interaction both in the chiral symmetry broken and restored regimes.

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**1. Introduction.** At moderate temperatures, below the critical one, the large  $N_c$  QCD is confining up to arbitrary large baryon densities [1]. This for a long time unexpected conclusion is based on a very simple and transparent argument. In the large  $N_c$  limit there are no dynamical vacuum quark loops and hence nothing screens a confining gluon propagator (whatever nature this propagator can be), i.e., in the strongly interacting dense matter gluodynamics is exactly the same as in vacuum and hence is confining. Consequently, it is possible to identify a new phase, called quarkyonic (which actually could consist of a few different "subphases"). In this dense but confined phase the bulk thermodynamic properties (like pressure) behave as  $O(N_c)$ , to be contrasted with the  $O(N_c^2)$  scaling in the high temperature deconfining phase and with the  $O(1)$  scaling in the low-temperature and density hadronic phase.

An interesting question arises. One typically expects that at some critical density in the strongly interacting matter spontaneously broken chiral symmetry of QCD should be restored, at least due to the Pauli blocking of the quark levels that are required for creation of a quark condensate. Then one arrives at a paradoxical situation: at finite density and low temperature one can expect existence of confined but chirally symmetric hadrons. According to the previous experience it was considered to be impossible. It has been demonstrated, however, that this is not so. It is possible to have manifestly chirally symmetric but confined hadrons above some critical density, at least within a model [2]. This model is manifestly confining and chirally symmetric and guarantees spontaneous breaking of chiral symmetry in the vacuum. The following mechanism for confining but chirally symmetric matter at large density is observed. Even though the Lorentz-scalar part of the quark self-energy vanishes in the chirally restored regime, there still exists the spatial Lorentz-vector self-energy. This self-energy is infrared-divergent and hence the single quark cannot be observed. In a color-singlet hadron this infrared divergence cancels exactly thus the color-singlet hadron is a finite and well-defined quantity. Consequently in this regime the hadron mass is generated from the manifestly confining and chirally-symmetric dynamics. If it is also a property of QCD (which is likely, but not yet proven), then in

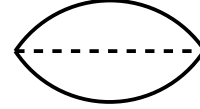


FIG. 1: Exchange contribution to the ground state energy.

the high density heavy ion collision studies one might see not a deconfining and chirally symmetric quark matter with single quark excitations, but rather a confining but chirally symmetric phase with the color-singlet hadronic excitations only.

The model relies on the linear,  $\sim 1/\bar{p}^4$ , Coulomb-like confining potential, that is indeed observed in the Coulomb-gauge QCD studies [3] as well as in Coulomb gauge lattice simulations [4]. Certainly the linear confining Coulomb-like potential alone does not represent a complete QCD Hamiltonian and some elements of the QCD dynamics are still missing. Nevertheless, such a simplified model allows one to answer some principal questions and obtain insight.

It is well-known, however, that energy of the ground state of a many-body nonrelativistic fermion system with the  $\sim 1/\bar{p}^4$  interaction is divergent already at the leading order in the interaction potential, due to graphs on Fig. 1. Consequently, an objection was posed that the quarkyonic matter with such an interaction should collapse and cannot exist [5]. In this report we address this issue and show that it is not so, because the infrared divergence of a pairwise confining force is exactly canceled by the infrared divergences of the single-quark self-energies in a color-singlet quarkyonic matter.

## 2. Confinement and chiral symmetry properties in a vacuum

Here we briefly overview some main elements of the model. For the last two decades this model has been exploited many times with different purposes, for some of the relevant references see [6] and references cited therein.

We work in the chiral limit and the two flavor version of the model is considered. The global chiral symmetry of the model is  $U(2)_L \times U(2)_R$  because in the large  $N_c$  world the axial anomaly is absent. The only interquark interaction in our case is a linear instantaneous Lorentz-vector potential that has a Lorentz structure of the Coulomb

$$\begin{aligned} \overline{S} &= \overline{S_0} + \overline{S_0} \text{ (self-energy)} + \overline{S_0} \text{ (two self-energies)} + \dots = \overline{S_0} + \overline{S_0} \text{ (self-energy)} \overline{S} \\ \text{---} \Sigma \text{---} &= \text{---} \text{ (gluon loop)} \text{---} + \text{---} \text{ (two gluon loops)} \text{---} + \dots = \text{---} \text{ (gluon loop)} \text{---} S \end{aligned}$$

FIG. 2: Dressed quark Green function and the Schwinger-Dyson equation.

potential:

$$K_{\mu\nu}^{ab}(\vec{x} - \vec{y}) = g_{\mu 0} g_{\nu 0} \delta^{ab} V(|\vec{x} - \vec{y}|); \quad \frac{\lambda^a \lambda^a}{4} V(r) = \sigma r, \quad (1)$$

where  $a, b$  are color indices. The Fourier transforms of this potential and any loop integral are infrared divergent. Hence to solve the gap (Schwinger-Dyson equation) and the bound state equations the infrared regularization is required. Any observable should not depend on the infrared cut-off parameter  $\mu_{IR}$  in the infrared limit, i.e., when this parameter is sent to 0,  $\mu_{IR} \rightarrow 0$ . There are several equivalent ways to perform a regularization. Here, specifically, we use the following regularized potential

$$V(\vec{p}) = \frac{8\pi\sigma}{(\vec{p}^2 + \mu_{IR}^2)^2}. \quad (2)$$

Then this potential in the configuration space contains the required  $\sigma r$  term, the infrared-divergent term  $-\sigma/\mu_{IR}$  as well as terms that vanish in the infrared limit.

Parametrizing the self-energy operator in the form

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma} \cdot \hat{\vec{p}})[B_p - p], \quad (3)$$

where functions  $A_p$  and  $B_p$  are yet to be found, the Schwinger-Dyson equation for the self-energy operator in the rainbow approximation, which is valid in the large  $N_c$  limit for the instantaneous interaction, see Fig. 2, is reduced to the nonlinear gap equation for the chiral (Bogoliubov) angle  $\varphi_p$ ,

$$A_p \cos \varphi_p - B_p \sin \varphi_p = 0, \quad (4)$$

where

$$A_p = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} V(\vec{p} - \vec{k}) \sin \varphi_k, \quad (5)$$

$$B_p = p + \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} (\hat{\vec{p}} \cdot \hat{\vec{k}}) V(\vec{p} - \vec{k}) \cos \varphi_k. \quad (6)$$

The functions  $A_p, B_p$ , i.e., the quark self-energy, are divergent in the infrared limit,

$$\begin{aligned} A_p &= \frac{\sigma}{2\mu_{IR}} \sin \varphi_p + A_p^f, \\ B_p &= \frac{\sigma}{2\mu_{IR}} \cos \varphi_p + B_p^f, \end{aligned} \quad (7)$$

where  $A_p^f$  and  $B_p^f$  are infrared-finite functions. This implies that the single quark cannot be observed and the system is confined. However, the infrared divergence cancels exactly in the gap equation (4) so this equation can be solved directly in the infrared limit. The chiral symmetry breaking is signaled by the nonzero chiral angle and quark condensate as well as by the dynamical momentum-dependent mass of quarks,  $M(p)$ .

Given a dressed quark Green function from the gap equation, one can solve the Bethe-Salpeter equation for mesons [7] or variational dynamical equations for baryons [8]. The infrared divergences cancel exactly in these equations for the color-singlet hadrons so the hadron mass is a well defined and finite quantity. [9]

### 3. Cancellation of the infrared divergences in quarkyonic matter

Before proving cancellation of the infrared divergences in the quarkyonic matter it is instructive first to recall how these divergences cancel in the color-singlet hadrons.

A (diverging) single-quark energy, i.e., a dispersive law, is determined by the single quark Green function and is given as

$$\omega(p) = \sqrt{A_p^2 + B_p^2} = \frac{\sigma}{2\mu_{IR}} + \omega_f(p), \quad (8)$$

where  $\omega_f(p)$  is an infrared-finite function. Note, that the infrared divergent term is not dependent on the chiral angle. This means that the nature of divergence is the same both in the Wigner-Weyl and Nambu-Goldstone modes. Then in the case of a meson bound state there are diverging contributions from the quark and the antiquark self-energies as well as from the quark-antiquark interaction potential. All these three contributions exactly cancel each other in the color-singlet  $\bar{q}q$  state,

$$2 \frac{\sigma}{2\mu_{IR}} - \frac{\sigma}{\mu_{IR}} = 0. \quad (9)$$

Consequently there are no infrared divergent contributions in the Bethe-Salpeter equation for  $\bar{q}q$  mesons [7]. An important issue is that the infrared divergent terms both in the quark self-energies and in the  $1/\vec{p}^4$  interquark potential do not depend on the absolute or relative coordinates of quarks. Consequently, the cancellation is complete and exact, whatever distance between the quarks is. This also means that in the infrared-divergent terms the color-dependent contributions factorize exactly.

The relative "-" sign between the self-energy divergences and the divergence from the quark-antiquark interaction kernel as well as equality of their absolute values is provided by the proper color-dependent Casimir factors. Indeed, the color factor for the quark (or antiquark) self-energy contribution is given by

$$\langle [1]_C | \frac{\lambda^a \lambda^a}{4} | [1]_C \rangle = 4/3, \quad (10)$$

while the quark-antiquark interaction color factor between the quark with the number "i" and the antiquark "j" is:

$$\langle \bar{q}_j q_i; [111]_C | - \frac{\lambda_i^a \lambda_j^{a*}}{4} | \bar{q}_j q_i; [111]_C \rangle = -4/3, \quad (11)$$

where  $[111]_C$  is a Young pattern for a color-singlet  $SU(3)_C$  color wave function. Note, that the factor  $4/3$  is included into  $\sigma$  in eq. (9), according to the definition (1).

In baryons the interquark interaction contribution contains a factor

$$\langle q^3; [111]_C | \frac{\sum_{i < j} \lambda_i^a \lambda_j^a}{4} | q^3; [111]_C \rangle = -2. \quad (12)$$

Hence one again observes a cancellation of the infrared divergences, because (12) cancels the factor (10) multiplied by the number of quarks in a baryon (do not forget additional factor  $1/2$  as seen in eq. (8) for the quark self-energies).

Now we are in a position to demonstrate a cancellation of the infrared divergences in a baryonic (quarkyonic) matter. We have shown such a cancellation in each color-singlet baryon. Hence, to prove such a cancellation in matter we only need to show that no new infrared divergences appear due to possible interaction between quarks belonging to different color-singlet 3q subsystems.

The interquark color interaction factor in an arbitrary system of  $n$  quarks is given by the quadratic Casimir operator  $C_2$  for  $SU(3)$ :

$$\frac{\sum_{i < j}^n \lambda_i^a \lambda_j^a}{4} = \frac{C_2}{2} - \frac{2n}{3}. \quad (13)$$

However, in a color-singlet many-quark system this Casimir operator is exactly zero,

$$C_2([color \ singlet]) = 0. \quad (14)$$

This means that in a color-singlet  $n$ -quark system the color-dependent factor is exactly the same as in a system of  $n/3$  color-singlet 3q clusters, which is not true in any color-non-singlet  $n$ -quark system. Consequently,

$$\langle [111]_C \times [111]_C \times \dots | \frac{\sum_{i \neq j} \lambda_i^a \lambda_j^a}{4} | [111]_C \times [111]_C \times \dots \rangle = 0, \quad (15)$$

where  $i$  and  $j$  belong to different color-singlet 3q subsystems. Cancellation is provided by the coordinate independence of the infrared-divergent terms. A complete antisymmetry of each color-singlet wave function in eq. (15) allows one to reduce this equation to a stronger one

$$\langle [111]_C \times [111]_C \times \dots | \frac{\lambda_i^a \lambda_j^a}{4} | [111]_C \times [111]_C \times \dots \rangle = 0, \quad (16)$$

for any  $i$  and  $j$  belonging to different color-singlet 3q subsystems. Consequently, no new infrared divergences appear from the interquark interaction in a matter, beyond those which exist in each color-singlet 3q subsystem. In each color-singlet 3q cluster such divergences cancel by the divergences from the quark self-energies. This proves exact cancellation of the infrared divergences in a dense matter, in particular in its ground state. Note that this cancellation equally applies to both the chiral symmetry broken phase as well as to the chirally symmetric quarkyonic matter.[10]

At this point we see a crucial difference between the quark matter in a color-singlet state and a many-fermion system without color. In the latter case the  $1/\bar{p}^4$  inter-fermion interaction leads to a divergence and such a system cannot exist. In the former case, however, a color-dependence of such a force provides an exact cancellation of the infrared divergences from the interquark interaction and quark self-energies. At the same time the infrared divergences persist in any non color-singlet system. This is consistent with the sufficient condition of confinement, namely that the color-singlet systems must be infrared finite, while all colored states must be infrared divergent.

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- [9] A choice of a proper infrared regularization is an important and subtle issue. As usual, a regularization prescription is a part of a *definition* of the Hamiltonian and different regularization prescriptions can imply different physical properties of the system. Then it is the physical requirements which determine a choice of a regularization scheme. In our case a constraint comes from the requirement of confinement - the physical spectrum should contain only the color-singlet states. The regularization (2) does satisfy this requirement. One could choose, however, another regularization prescription, e.g.,  $\sigma r \rightarrow \sigma r \exp(-\mu_{IR} r)$ , that was also used in the literature in the past. This prescription leads to the same gap equation as well as to the same spectrum of the color-singlet hadrons. However, with this prescription the single-quark energy is infrared-finite. This means that the spectrum of the such defined theory contains both the color-singlet hadrons and the colored single-quark states. Obviously it does not satisfy the requirement of confinement and only those regularization prescriptions can be used that remove single quarks from the spectrum.
- [10] In a dense matter the infrared finite parts  $A_p^f$  and  $B_p^f$  of the quark self-energy are of course not the same as in the vacuum because we have to exclude all occupied intermediate levels with  $\vec{k} \neq \vec{p}$  - they cannot contribute due to Pauli blocking. The infrared-divergent part of the self-energy is, however, exactly the same as in the vacuum, because the divergent contribution comes from the point  $\vec{k} = \vec{p}$  and the Pauli blocking of the other levels does not affect this point.